Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WFM02/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Further Pure Mathematics F2

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶


©2021 Pearson Education Ltd.
E:1/1/1/

P 6 9 1 9 3 A 0 1 3 6

Pearson

Leave blank

**1.** Solve the equation

$$z^5 - 32\mathrm{i} = 0$$

giving each answer in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $0 < \theta < 2\pi$

**(4)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 1 continued**

**Q1**

**(Total 4 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**2.** Use algebra to determine the set of values of $x$ for which

$$\frac{x}{2-x} \leqslant \frac{x+3}{x}$$

(*Solutions relying entirely on graphical methods are not acceptable.*)

**(8)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 2 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 2 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

**Question 2 continued**

**Q2**

**(Total 8 marks)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

**3.** A transformation maps points from the $z$-plane, where $z = x + \mathrm{i}y$, to the $w$-plane, where $w = u + \mathrm{i}v$. The transformation is given by

$$w = \frac{(2 + \mathrm{i})z + 4}{z - \mathrm{i}} \qquad z \neq \mathrm{i}$$

The transformation maps the imaginary axis in the $z$-plane onto the line $l$ in the $w$-plane.

Determine a Cartesian equation of $l$, giving your answer in the form $au + bv + c = 0$ where $a$, $b$ and $c$ are integers to be found.

**(6)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 3 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 3 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

**Question 3 continued**

**Q3**

**(Total 6 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**4.** (a) Determine the general solution of the differential equation

$$(x+1)\frac{dy}{dx} - xy = e^{3x} \qquad x > -1$$

giving your answer in the form $y = f(x)$.

**(7)**

(b) Determine the particular solution of the differential equation for which $y = 5$ when $x = 0$

**(2)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 4 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 4 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 4 continued**

**Q4**

**(Total 9 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**5.** Given that $y = \tan^2 x$

(a) show that

$$\frac{d^3y}{dx^3} = 8\tan x \sec^2 x\left(p\sec^2 x + q\right)$$

where $p$ and $q$ are integers to be determined.

**(5)**

(b) Hence determine the Taylor series expansion about $\frac{\pi}{3}$ of $\tan^2 x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$, giving each coefficient in simplest form.

**(3)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 5 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 5 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 5 continued**

**Q5**

**(Total 8 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**6.** The complex number $z$ on an Argand diagram is represented by the point $P$ where

$$|z + 1 - 13\mathrm{i}| = 3|z - 7 - 5\mathrm{i}|$$

Given that the locus of $P$ is a circle,

(a) determine the centre and radius of this circle.

**(5)**

The complex number $w$, on the same Argand diagram, is represented by the point $Q$, where

$$\arg(w - 8 - 6\mathrm{i}) = -\frac{3\pi}{4}$$

Given that the locus of $P$ intersects the locus of $Q$ at the point $R$,

(b) determine the complex number representing $R$.

**(4)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 6 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 6 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

**Question 6 continued**

**Q6**

**(Total 9 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**7.** (a) Show that the transformation $x = t^2$ transforms the differential equation

$$4x\frac{d^2y}{dx^2} + 2\left(1 + 2\sqrt{x}\right)\frac{dy}{dx} - 15y = 15x \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 15y = 15t^2 \qquad \text{(II)}$$

**(5)**

(b) Solve differential equation (II) to determine $y$ in terms of $t$.

**(5)**

(c) Hence determine the general solution of differential equation (I).

**(1)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 7 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 7 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 7 continued**

**(Total 11 marks)**

Q7

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**8.**

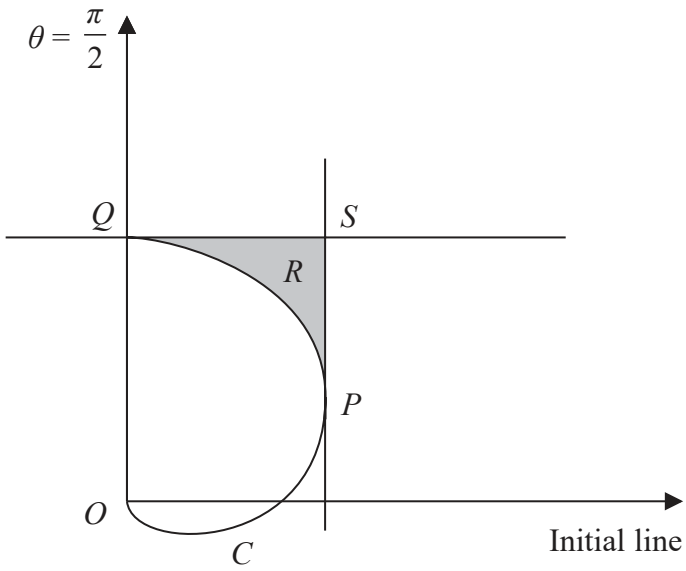


**Figure 1**

The curve $C$ shown in Figure 1 has polar equation

$$r = 1 + \sin\theta \qquad -\frac{\pi}{2} < \theta \leqslant \frac{\pi}{2}$$

The point $P$ lies on $C$ such that the tangent to $C$ at $P$ is perpendicular to the initial line.

(a) Use calculus to determine the polar coordinates of $P$.

**(5)**

The tangent to $C$ at the point $Q$ where $\theta = \frac{\pi}{2}$ is parallel to the initial line.

The tangent to $C$ at $Q$ meets the tangent to $C$ at $P$ at the point $S$, as shown in Figure 1.

The finite region $R$, shown shaded in Figure 1, is bounded by the line segments $QS$, $SP$ and the curve $C$.

(b) Use algebraic integration to show that the area of $R$ is

$$\frac{1}{32}\left(a\sqrt{3} + b\pi\right)$$

where $a$ and $b$ are integers to be determined.

**(6)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 8 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 8 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 8 continued**

**(Total 11 marks)**

**Q8**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

**9.** (a) Show that

$$n^5 - (n-1)^5 \equiv 5n^4 - 10n^3 + 10n^2 - 5n + 1$$

**(2)**

(b) Hence, using the method of differences, show that for all integer values of $n$,

$$\sum_{r=1}^{n} r^4 = \frac{1}{30} n(n+1)(2n+1)(an^2 + bn + c)$$

where $a$, $b$ and $c$ are integers to be determined.

**(7)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 9 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 9 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 9 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

**Question 9 continued**

Q9

**(Total 9 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 6 9 1 9 3 A 0 3 6 3 6